



H_{∞} Filter with Fuzzy Logic Estimation Based SLAM to Refrain Finite Escape Time: An Analysis in Different Mobile Robot Movement

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ARTICLE INFO

Article history:

Available online June, 12 2022

Keywords:

mobile robot

H_{∞} filter (HF)

finite escape time (FET)

fuzzy logic

simultaneous localization and mapping (SLAM)

ABSTRACT

In this paper, a mobile robot simultaneous localization and mapping (SLAM) in non-Gaussian noise environment is considered. Over pass decade, the famous Extended Kalman Filter (EKF) are aggressively being used in mobile robot based SLAM observation, but its capabilities only limited in Gaussian noise environment. Since the study is to emphasize the application of autonomous mobile robot in a real life application, the environment is imprecise. Despite to this limitation, H_{∞} Filter (HF) being choose that may provide better solution in non-Gaussian noise environment. However, HF suffer with Finite Escape Time (FET) issue that limits the HF estimation capabilities and may lead to inaccurate estimation result. Hence, in order to pursue the best performance of SLAM, a new H_{∞} Filter with Fuzzy Logic (FHF) is proposed. A new FHF technique is developed by adding a fuzzy logic rules and fuzzy set in HF innovation stage. The proposed technique applies the information extracted from the HF measurement innovation. The investigation is done in triangular membership. With suitable range of each membership produces the simulation result convinces that FHF effectively capable in reducing the FET from occurring in mobile robot localization and simultaneously improve the estimation between mobile robot and landmarks.

1. Introduction

For decades, the development of mobile robots in navigation systems has attracted more researchers' attention due to its broad application prospects. They are trying to build an autonomous mobile robot that can discover and recognize unknown environments. Due to the limitations of humans working in hazardous areas, it is very useful to know the position and orientation of mobile robots in different tasks and environments, which can be seen in multiple studies (Thrun, Burgard, & Fox, 2000, 2005; Yang, Wang, Lauria, & Liu, 2009; S. Zhang et al., 2016).

A variety of mobile robot positioning methods have been developed in the past. The main difference lies in the technology used to express the robot's belief in its current location. Smith and Cheeseman first proposed Synchronous Localization and Mapping (SLAM) in 1987 as one of the technologies for mobile robots to locate themselves and gradually build maps. The development and progress of SLAM continue to inspired influence from series of work done by R. Smith et al. and Hugh Durrant Whyte et al since 1990's (Durrant-Whyte, 1988; R. C. Smith & Cheeseman, 1986).

Most solutions to the SLAM problem aim to create high-resolution maps of the observed area. The efficiency with which a mobile robot can accomplish a given task in SLAM depends on the robot's positioning and sensors. Interest in solving SLAM problems has grown rapidly and excitingly over the past decade.

SLAM involves the observation process performed by a mobile robot in a given area, while creating a map as it moves in an unknown area, in meantime keep tracking to determine its own position on the map. (Montemerlo, Thrun, Koller, & Wegbreit, 2002; Thrun et al., 2000). Most solutions to the SLAM problem aim to create high- resolution maps of observed area. The efficiency with which a mobile robot can accomplish a given task in SLAM depends on the robot's positioning and sensors. Interest in solving SLAM problems has grown rapidly and excitingly over the past decade. Various approach on SLAM being done in different ways (Asadi & Bozorg, 2009; Huang & Dissanayake, 2007) both theoretical and practical, but they still facing a lot of unsolved issue that effect the estimation process (Hamzah Ahmad & Namerikawa, 2013; Huang & Dissanayake, 2007).

2. Literature Review

The development of autonomous mobile robots increased drastically significant to the variety range of new applications of mobile robot. Today, mobile robot can be found in various application due to the continuous development and improvement on autonomous navigation system such as indoor, outdoor, outer space, underwater, industrial production, households, education environment and etc. (Hähnel, Burgard, & Thurn, 2003; Ratter & Sammut, 2015; Thrun et al., 2000; Tobata, Kurazume, Iwashita, & Hasegawa, 2010; Yang et al., 2009). Knowing the position and orientation of a mobile robot is very useful in different tasks and environments (Yang et al., 2009; S. Zhang et al., 2016) and the expanding application of SLAM is expected to be boosting up in future.

However, two key problems that need to be addressed by autonomous mobile robots to fully functional are navigate safely through the unknown environments without any human interference and successfully performing design tasks at the same time. The mobile robot must capable to map the unknown environment and finds its location within the map simultaneously. When dealing with mapping and robot localization simultaneously, it will involve four major problems that need to be considered which are map establishment, self-localization, path detection and obstacle avoidance, each of these problem are correlated to each other (Lin & Chen, 2011). The dependency between robot pose and the map estimation makes the SLAM problem difficult and requires further research for better solution.

In response to this problem, a lot of research is being done to improve the efficiency and quality of SLAM. The goal of all work is maps that are highly accurate in terms of accuracy, estimation, and reduced error rates. Most methods for SLAM focus on probabilistic Bayesian Estimation, such as Kalman Filter (KF), Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), Particle Filter (PF) and etc., but the most famous method is EKF that being introduce in 1988 by (R. Smith, Self, & Cheeseman, 1988). The effectiveness of EKF methods is based on the facts that they can estimate fully correlated posteriors across landmarks and robot poses, which makes them to become most popular and dominant technique used in SLAM.

Although EKF used immensely in various approaches (Bakar, Hm, & Lotfy, 2016; Ganesha Perumal, Subathra, Saravanakumar, & Srinivasan, 2016; Hammia & Team, 2020; Paz & Neira, 2006; Rongchuan, Shugen, Bin, & Yuechao, 2008; Ullah, Su, Zhang, & Choi, 2020; F. Zhang, Li, Yuan, Sun, & Zhao, 2017), the performance of the EKF is limited to Gaussian noise conditions, where the EKF requires statistical information about the input noise, which is often imprecise in practical robotics applications, which limits its practical application. For practical applications in real environments, non-Gaussian noise environments must be further considered in SLAM.

Based on previous work, another filter named Particle Filter (PF) being introduce by (Liu & Chen, 1998), this filter also known as Sequential Monte Carlo (SMC) and seem suitable for the state estimation in non-linear problem and applicable in non-Gaussian system and gradually being applied in SLAM (Agand, Taghirad, & Khaki-Sedigh, 2017; Dai, Xu, & Li, 2018; Sun, Hou, Zhang, & She, 2017; Zhang, Y., Zheng, Y. F., & Luo, 2016). It seems to be convincing among others filter and capable to work in non-Gaussian noise condition. However, in order to gain high precision estimation, PF requires a large number of particles thus will increase the computational complexity and precision.

2.1 Problem statement

Due to these circumstances, based on previous work, another filter name as H_{∞} Filter (HF) may provide better solutions (Hamzah Ahmad & Namerikawa, 2011a, 2011b). HF outperforms KF, providing better estimates in non-Gaussian noise environments (Hamzah Ahmad & Othman, 2015a). Several works are also carried out in Hamzah Ahmad & Namerikawa, 2011b; Hamzah Ahmad & Othman, 2015b; West & Syrmos, 2006 to further investigate the performance of HF, and the results show that HF gives more reliable estimates, is more robust, and is more suitable for practice. HF does not require any statistical noise assumptions.

SLAM APPROACH	KALMAN FILTER (KF)	H_{∞} FILTER (HF)	PARTICLE FILTER (PF)
Noise	Gaussian	Gaussian & non-Gaussian	Gaussian & non-Gaussian
Advantage	Lower computation cost, Robust	Capable to work in non-Gaussian noise, Uncertain and robust condition, more reliable estimation	Accurate reading
Difficulties	Non-Gaussian (Need further considerations about the environment)	Finite Escape Time (FET), γ selection	Complex, High Computation Cost, Offline application

Table 1.1: Comparison of main Approach in SLAM

According to the comparison in Table 1.1, although the performance of HF is promising, HF still suffers from the finite escape time (FET) problem during mobile robot navigation. The FET phenomenon is unavoidable in ordinary HF and becomes one of the factors that lead to filter inconsistency, making HF less popular than KF or EKF, despite HF's proficiency in non-Gaussian noise. A FET is a state in which the state covariance with the normal state becomes positive or negative infinity, which can lead to erroneous results.

There are some research done that focused on HF based SLAM but not highlighted on solving FET issue until in 2011, limited research attempted to analyzed the conditions of FET in HF and gave the positive finding. Starting with Hamzah Ahmad & Namerikawa, 2011b that required smaller γ value, follow by (Okawa & Namerikawa, 2013) that proposed using priori-known-landmark H_∞ Filter algorithm. The error covariance matrix is suppressed and proved capable of avoided FET. While in (Othman, Ahmad, & Namerikawa, 2015) proposed method where the measurement noise must be less than γ^2 . In 2016, another method of adding pseudo PSD to the state covariance to avoid FET proposed by (H. Ahmad & Othman, 2016), the result is still influenced the estimation and produced high amount of uncertainties.

Due to this, a new method of combining HF with fuzzy logic to create H_∞ Filter with Fuzzy Logic (FHF) proposed by (Hamzah Ahmad & Othman, 2015a). The research done by using fuzzy logic in Gaussian membership. This work done encourage by lot of previous study that show a successful result in combining EKF with Fuzzy Logic for better performance (Choomuang & Afzulpurkar, 2005). While in 1995, (Kobayashi, Cheok, & Watanabe, 1995) found that fuzzy logic was able to tune the covariance and reset the initialization of the filter. The FHF method finally able to preserve the FET from happening during estimation. FHF prove to perform better and improve the performance of HF especially when the FET is being considered. Since the analysis done by (Hamzah Ahmad & Othman, 2015a) using the Fuzzy Logic with Gaussian membership function, others membership is still not being examine yet.

In Bakiss Hiyana Abu Bakar & Ahmad (2021), further analysis in FHF using triangular membership being proposed and result show that it's also capable to refrain the present of FET compare to HF. Encourage by this reason, throughout this research, further analysis will be done using FHF in triangular membership but in different mobile robot movement to prove FHF with triangular membership capable to refrain the present of FET and give better estimation result compare to HF.

3. Proposed Method

Compared with Kalman filter (KF) or EKF, although HF is competent in non-Gaussian noise, FET phenomenon is unavoidable in ordinary HF, which makes this filter less popular. The presence of field-effect transistors will cause the state covariance of the mobile robot to become positive or negative during the observation process, even after recording multiple measurements of the surrounding environment, it will also cause the robot to lose confidence in determining the position.

To overcome the FET problem and obtain a high-precision map, our key method is to control the measurement input of the mobile robot. This article will compare the estimated measurement of HF with the measurement information through measurement innovation. If the robot observes a large error during the observation, it will affect the estimation and cause error. To reduce errors, measurement innovation must always be kept small to improve estimates. Further details will be discussed in the next sub-section.

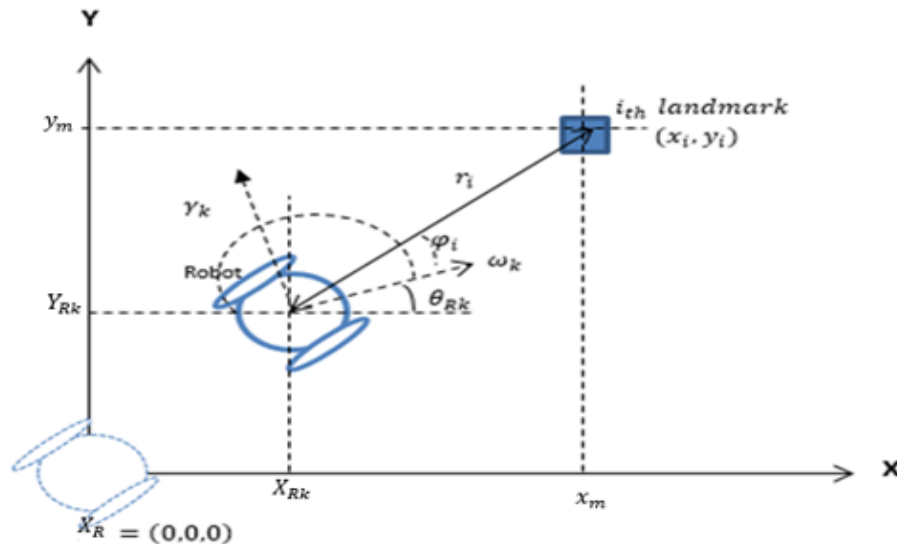


Figure 1.1: Prediction and Measurement Process Model

3.1 SLAM General model

The localization and mapping of mobile robots requires two processes, namely, the process model and the measurement model. The process model calculates the kinematics of the mobile robot, while the measurement model measures the distance between the robot and landmarks during observation. The process model of mobile robot describes as follow:

$$X_{(k+1)} = f(X_k, \omega_k, \varphi_k, \delta_\omega, \delta_\varphi) \quad [1]$$

Where X_k is state vector that consist the x, y positions and heading angle θ_k of a mobile robot. ω_k, φ_k is the angular acceleration and the velocity of the mobile robot with its associated noise $\delta_\omega, \delta_\varphi$. The state vector of SLAM robot is a combination between robot position X_r and landmark position X_m as follow structure:

$$X_k = [X_r \ X_m]^T \quad [2]$$

Where the position of mobile robot is $X_r = [X_{Rk} \ Y_{Rk} \ \theta_{Rk}]^T$ is represent the center of mobile robot coordinate ($X_{Rk} \ Y_{Rk}$) and the heading angle θ_{Rk} . The state of the landmarks is represented by $X_m = [l_1 l_2 \dots l_m]^T$. The landmarks is describe as the coordinate ($x_i \ y_i$) with $i = 1, 2, 3, \dots, m$, where m refer to the number landmarks.

The measurement model of mobile robot defined as follow:

$$Z_k = \begin{bmatrix} r_k \\ \varphi_k \end{bmatrix} = [H_k X_k + V_{r,k}] = \begin{bmatrix} \sqrt{(x_i - x_{Rk})^2 + (y_i - y_{Rk})^2} + v_{r,k} \\ \tan^{-1} \left(\frac{y_i - y_{Rk}}{x_i - x_{Rk}} \right) - \theta_{Rk} + v_\varphi \end{bmatrix} \quad [3]$$

Where Z_k is the sensor reading of the robot that represent the relative distance and angle measurement, r_k and φ_k among mobile robot and landmark, H_k represent the measurement matrix while $v_{r,k}$ and v_φ are the measurement noise that will affect the measurement reading.

3.1.1 H_∞ Filter algorithm

H_∞ Filter based SLAM being proposed by (West & Syrmos, 2006). H_∞ Filter (HF) algorithm is almost similar to the structure of Extended Kalman Filter (EKF) based SLAM. The different between those techniques is only on the presence of γ in the state error covariance. HF generally involves two basic stage which are prediction and update which provide necessary information about the surroundings during the mobile robot observation process. In this research, we consider a nonlinear system to describe the problem explicitly.

HF has two recursive stages which are prediction and update stages as illustrate in Figure 1. During the mobile robot movements, the measurement of i -th landmark obtained using sensor on board of the robot is given by ω_k, φ_k , which are velocity and angular acceleration respectively. The measurement model calculates and measured the distance between robot and landmarks. The prediction stage is defined as follow.

$$\hat{X}_{k+1}^- = f_k(\hat{X}_k, \omega_k, \varphi_k, 0, 0) \quad [4]$$

$$P_{k+1}^- = f_k P_k (I - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k)^{-1} f_k^T + Q_k \quad [5]$$

Equation [4] defines the state prediction while equation [5] describe the state covariance. I in equation [5] represent the identity matrix, R is the measurement covariance and Q_k is the control noise covariance. For the update stage, it is defined as follow.

$$\hat{X}_{k+1} = f_k \hat{X}_k + f_k K_k (z_k - H_k \hat{X}_k) \quad [6]$$

$$K_k = P_k (I - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k)^{-1} H_k^T R_k^{-1} \quad [7]$$

Where the gain of the system is K_k . The second right term from equation [6] shows the measurement innovation that indicates the error of the system. The error is preferable to be small at all time or else the whole estimation will become erroneous. Due to this aspect, by controlling the size of covariance eventually contributes to control the size of error as at the same time could possibly control the present of FET as being done in (Bakiss Hiyana Abu Bakar & Ahmad, 2021) the research aims to decrease the error generated by the second term of equation [6] from getting bigger and at the same time controlling the present of FET.

3.1.2 H_∞ Filter with fuzzy logic design

In this study, we consider designing a two-input two-output fuzzy system that takes angle and distance errors as inputs. The main purpose of adding fuzzy logic to a system is to reduce errors by using fuzzy set configuration. By choosing the correct output, measurement errors caused by sensor inaccuracy can be minimized and further reduced.

The proposed fuzzy logic design is analyzed using the Mamdani technique, which is proposed because it analyzes and computes the output by considering all the information obtained from the measurements. Figure 1.2 shows the traditional of HF measurement innovation, while Figure 1.3 show the HF measurement innovation design with the application of Fuzzy Logic.

This technique is proposed because the method is able to analyse and calculate the output by considering all information obtained from the measurement. Since HF is very competence in non-Gaussian noise environment, the sensor reading may be interrupted, indicating large errors, and thus resulting in large measurement noise covariance, R . Smaller measurement errors are the key to better filter performance. This will only possible if the value of gain K is always small.

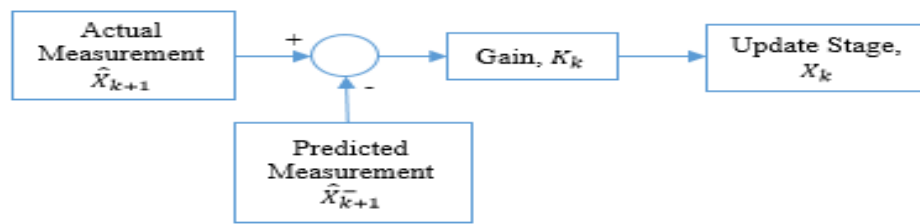


Figure 1.2: Conventional HF measurement

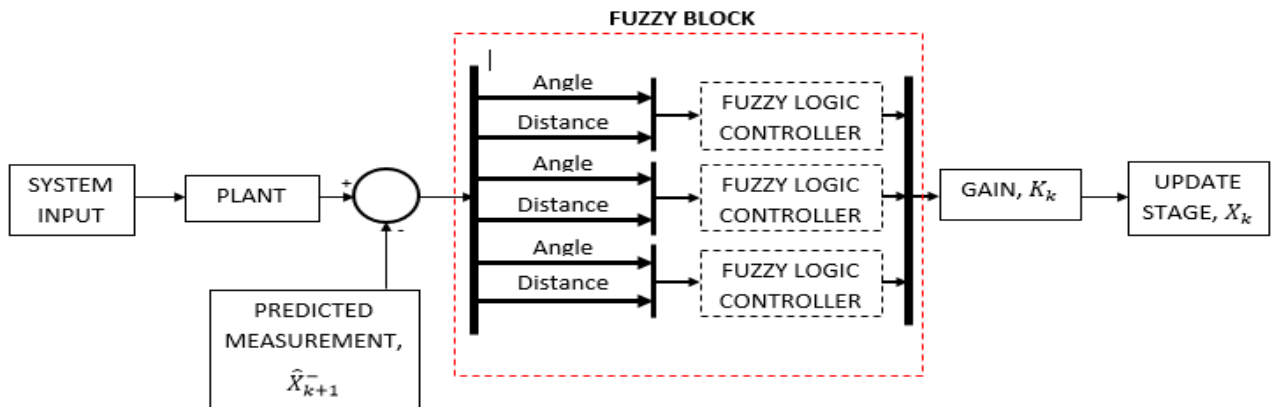


Figure 1.3: HF measurement innovation: Proposed Method of FHF for Mobile Robot SLAM

Motivate by this, the fuzzy logic controller is applied before gain K_k in order to control the size of the covariance by controlling the measurement input of each landmark. The research attempts to reduce the error generated by the second right term of equation [6] if the calculation suddenly generates higher value and finally result in FET. The gain K_k depends on the numbers of landmarks as well as the size of matrix defined by equation [7].

For evaluation purpose in this paper, triangular membership is considered. Three fuzzy set being choose with three categories, which are negative, positive and normal regions. Each fuzzy set being tune several times in order to obtain the best estimation result. Since the mobile robot in global coordinate move randomly, all positive and negative value is considered. This approach different from (Bakiss Hiyana Abu Bakar & Ahmad, 2021) since the mobile robot movement input are different. To define the measurement innovation, the fuzzy logic rules is describe as in table 1.2.

IF	THEN
Angle error is negative AND distance error is negative	Angle is negative
Angle error is negative AND distance error is normal	Angle is normal
Angle error is negative AND distance error is positive	Angle is negative, Distance is normal
Angle error is positive AND distance error is normal	Angle is positive
Angle error is positive AND distance error negative	Distance is normal
Angle error is positive AND distance error is positive	Angle is positive, Distance is normal

Table 1.2: Fuzzy Logic Rules

3.1.3 Simulation parameters and assumption

The first hypothesis involves mobile robot. The mobile robot involve in this simulation is a two-wheeled mobile robot. The robot assumes that it starts from a global coordinate system of (0,0). The robot move randomly in the environment and detects any nearby landmarks and measured the data to feed into FHF for analysis and updates state. All landmarks are defined as point landmarks, each of which is located at a specific location. This assumption was made to determine whether the technique could overcome the FET problem which can exist under defined and unknown boundary conditions.

The next assumption involves data association. Data association expected should always be available. In this study, the mobile robot has some environmental information defined by the covariance P at the initial stage. This assumption is important for overcoming the "hijacked robot problem," which means that a robot can be accidentally lost through its position.

The most important assumptions are related to the γ value. This value leads to the performance of the estimated result. The γ value was chosen based on (H. Ahmad & Othman, 2016) as the benchmark as it has been shown to be the best estimate. Finally, since the purpose of proposing HF for the SLAM problem is to treat the simulation as a non-Gaussian noise condition, so this simulation will be considered in this noise condition.

Variable	Parameter Value
PROCESS NOISE, Q_{min}, Q_{max}	-0.002, 0.001
MEASUREMENT NOISE, $R_{\theta_{min}}, R_{\theta_{max}}$ $R_{dist_{min}}, R_{dist_{max}}$	-0.04, 0.01 -0.15, 0.3
INITIAL COVARIANCE, $P_{robot}, P_{landmark}$	0.0001, 100
SIMULATION TIME (S)	1000s
GAMMA VALUE (γ)	0.7

Table 1.3: Simulation Parameters

For good consistency of simulation results, all simulations performed by MATLAB Simulink were performed within 1000 [s] and used parameters in table 2 with selected initial conditions. It is expected that the mobile robot will be equipped with at least one measurement sensor.

4. Results and Discussion

An analysis being done by (Bakiss Hiyana Abu Bakar & Ahmad, 2021) proved that after tuning process, FHF constructed better map than HF and illustrates less error for both robot and landmarks estimations. Mobile robot state covariance in FHF demonstrated that the state error covariance gradually decrease and much consistence in the estimation compare to HF.

Further analyses are conducted using different robot motions for further estimation. These aims are to observe FHF-based SLAM with triangular membership performance and to prove the ability of the proposed method to suppress FETs in HF. The proposed method is being tested in two different mobile robot motions, and the results are shown below.

4.1 FHF based SLAM Technique using Triangular Membership: Different Movement 1

The analysis on different movement 1 in triangular membership is done to observe the estimation on mobile robot movement and covariance when the robot move near to the landmarks area in wide dimension.

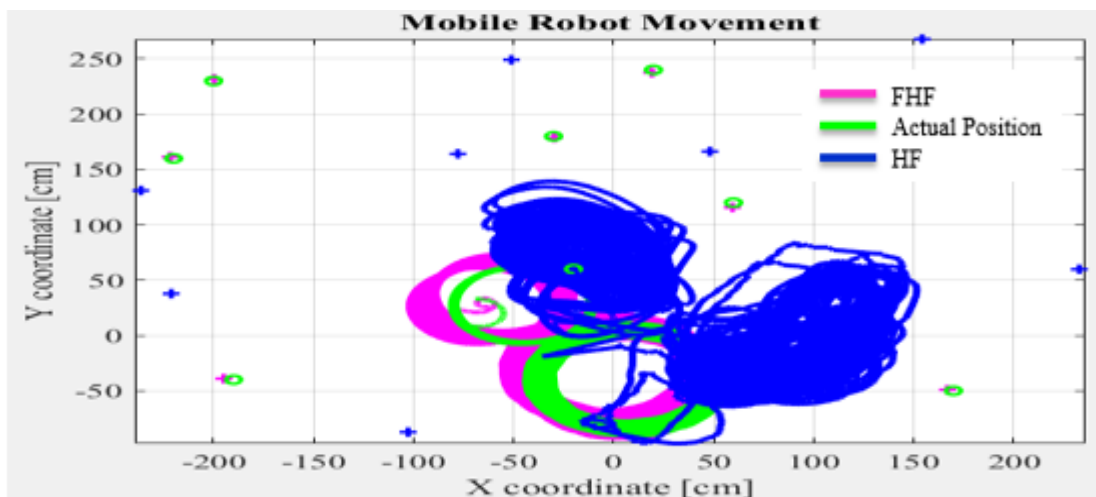


Figure 1.4: Different Movement 1 in triangular membership: SLAM performance between original HF (blue) & FHF (magenta) based on actual position (green) through environment. FHF show consistent reading in both robot path and the landmarks.

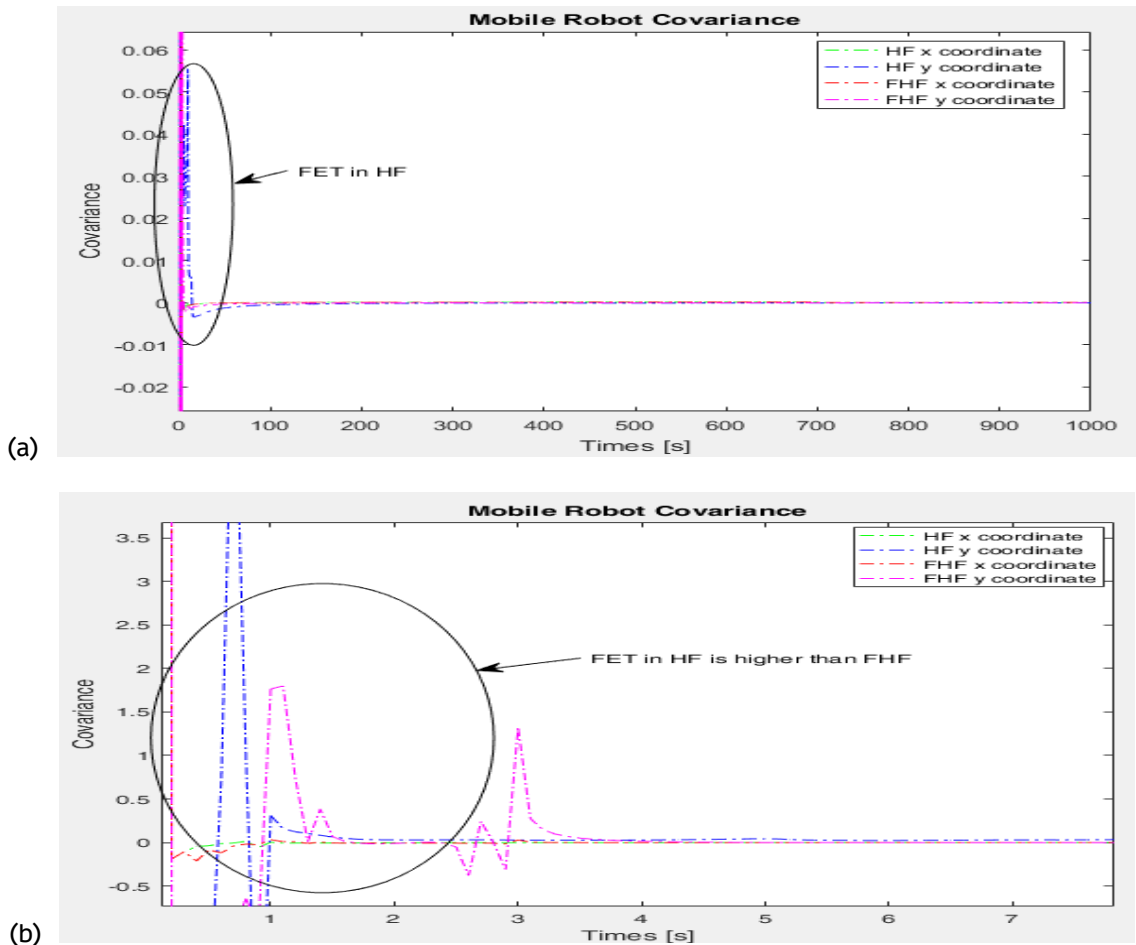


Figure 1.5: Different Movement 1 in triangular membership: The State Covariance between original HF and FHF. Both filters states subsequently goes infinite in initial time. But FHF show better performance compare to HF. (a) Overall 1000s result (b) Close Up to initial state.

The result likewise show much better estimation result both in term of robot movement and covariance in FHF compare to HF as illustrated in Figure 1.4 and Figure 1.5. These strongly proved that the H_∞ Filter with fuzzy logic in triangular membership also capable to refrain the existence of FET. To emphasis the proposed method of H_∞ Filter with fuzzy logic in triangular membership can refrain the existence of FET, another analysis is done to test the method in others types of mobile robot movement as show in Figure 1.6 below.

4.2 FHF based SLAM Technique using Triangular Membership: Different Movement 2

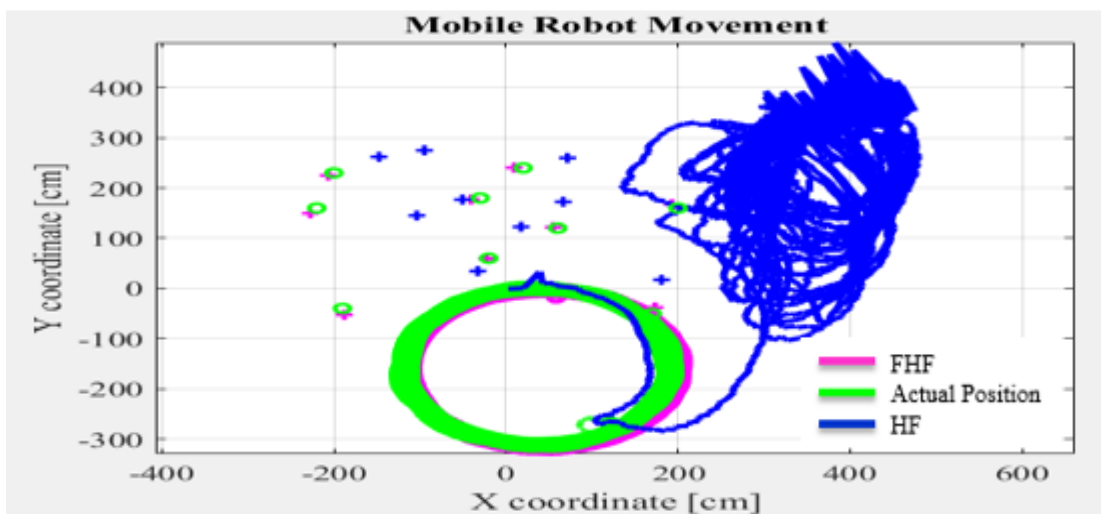


Figure 1.6: Different Movement 2 in triangular membership: SLAM performance between original HF (blue) and FHF (magenta) through Environment. The filter performance are compared based on actual position (green). FHF show better performance compare to HF.

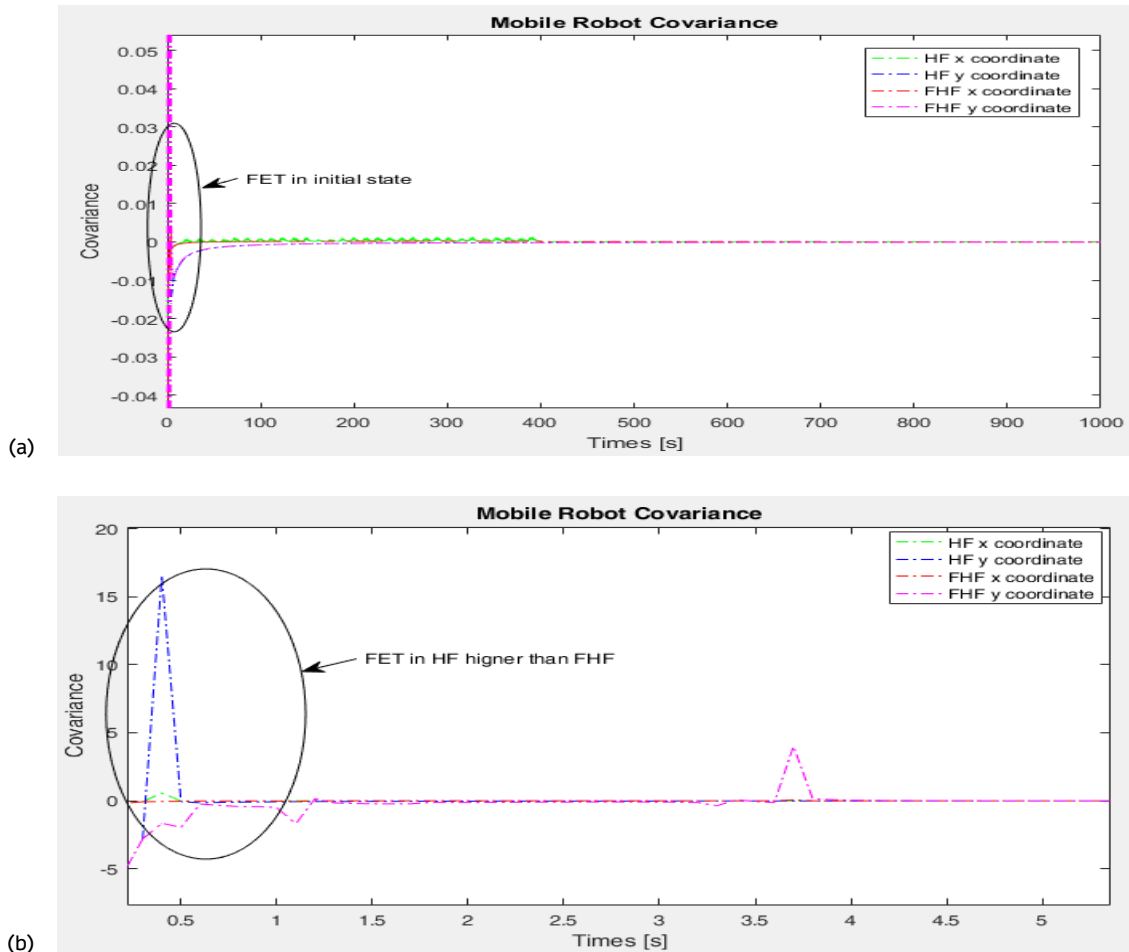


Figure 1.7: Different Movement 2 in triangular membership: The State Covariance between original HF and FHF. Both filters states subsequently goes infinite in initial time. (a) Overall 1000s result (b) Close Up to initial state.

Thus estimation result again consequently prove that H^∞ Filter with fuzzy logic in triangular membership capable to refrain the present of FET in HF FET and can be used as a solution for SLAM and likewise give better performance in mobile robot map construction compare to conventional H^∞ Filter.

5. Conclusion

As the main objective is to refrain the FET, the proposed technique is relevant and prove able to overcome the issue. Thus result clearly defines that the effect of suitable setting on fuzzy logic membership is crucial and has significant effect to the performance of FHF. The triangular membership in H^∞ Filter with fuzzy logic in capable to refrain the present of FET in HF and can be used as a solution for SLAM and likewise give better performance in mobile robot map construction compare to conventional H^∞ Filter.

Acknowledgment

The author would like to thanks Politeknik Sultan Haji Ahmad Shah (POLISAS) for continuous support in realizing this research. Special thanks also for Prof. Madya Ts. Dr. Hamzah bin Ahmad, from University Malaysia Pahang, Malaysia, for his full support and guidance in completing this research.

References

- Agand, P., Taghirad, H. D., & Khaki-Sedigh, A. (2017). Particle filters for non-Gaussian hunt-crossley model of environment in bilateral teleoperation. 4th RSI International Conference on Robotics and Mechatronics, ICRoM 2016, 512–517. <https://doi.org/10.1109/ICRoM.2016.7886794>
- Ahmad, H., & Othman, N. A. (2016). A solution to finite escape time for H^∞ filter based SLAM. Journal of Telecommunication, Electronic and Computer Engineering, 8(11), 7–13.

- Ahmad, Hamzah, & Namerikawa, T. (2011a). H_{∞} Filter-SLAM: A sufficient condition for estimation. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 44(1 PART 1), 3159–3164. <https://doi.org/10.3182/20110828-6-IT-1002.00260>
- Ahmad, Hamzah, & Namerikawa, T. (2011b). Robotic Mapping and Localization Considering Unknown Noise Statistics. *Journal of System Design and Dynamics*, 5(1), 70–82. <https://doi.org/10.1299/jsdd.5.70>
- Ahmad, Hamzah, & Namerikawa, T. (2013). Extended Kalman filter-based mobile robot localization with intermittent measurements. *Systems Science and Control Engineering*, 1(1), 113–126. <https://doi.org/10.1080/21642583.2013.864249>
- Ahmad, Hamzah, & Othman, N. A. (2015a). HF-fuzzy logic based mobile robot navigation: A solution to Finite Escape Time. *ARPN Journal of Engineering and Applied Sciences*, 10(23), 17559–17565.
- Ahmad, Hamzah, & Othman, N. A. (2015b). The impact of cross-correlation on mobile robot localization. *International Journal of Control, Automation and Systems Volume*, 13, 1251–1261. <https://doi.org/10.1007/s12555-014-0076-6>
- Asadi, E., & Bozorg, M. (2009). A decentralized architecture for simultaneous localization and mapping. *IEEE/ASME Transactions on Mechatronics*. <https://doi.org/10.1109/TMECH.2008.2009309>
- Bakar, A., Hm, S., & Lotfy, A. H. (2016). An Implementation of SLAM with Extended Kalman Filter, 2–5.
- Bakiss Hiyana Abu Bakar, & Ahmad, H. (2021). H_{∞} Filter with Fuzzy Logic Estimation to Refrain Finite Escape Time. *Recent Trends in Mechatronics Towards Industry 4.0, Lecture Notes in Electrical Engineering*, 730, 303–312. https://doi.org/10.1007/978-981-33-4597-3_28
- Choomuang, R., & Afzulpurkar, N. (2005). Hybrid Kalman filter/Fuzzy logic based position control of autonomous mobile robot. *International Journal of Advanced Robotic Systems*, 2(3), 197–208. <https://doi.org/10.5772/5789>
- Dai, J. H., Xu, P. C., & Li, X. B. (2018). Second order central difference particle filter FastSLAM algorithm. *Kongzhi Lilun Yu Yingyong/Control Theory and Applications*. <https://doi.org/10.7641/CTA.2018.60849>
- Durrant-Whyte, H. F. (1988). Uncertain Geometry in Robotics. *IEEE Journal on Robotics and Automation*. <https://doi.org/10.1109/56.768>
- Ganesha Perumal, D., Subathra, B., Saravanakumar, G., & Srinivasan, S. (2016). Extended kalman filter based path-planning algorithm for autonomous vehicles with I2V communication. *IFAC-PapersOnLine*, 49(1), 652–657. <https://doi.org/10.1016/j.ifacol.2016.03.130>
- Hähnel, D., Burgard, W., & Thrun, S. (2003). Learning compact 3D models of indoor and outdoor environments with a mobile robot. *In Robotics and Autonomous Systems*. [https://doi.org/10.1016/S0921-8890\(03\)00007-1](https://doi.org/10.1016/S0921-8890(03)00007-1)
- Hammia, S., & Team, T. I. M. (2020). Novel Hardware Architecture of EKF-SLAM' s Jacobian matrices and its FPGA Implementation, 1–5.
- Huang, S., & Dissanayake, G. (2007). Convergence and consistency analysis for extended Kalman filter based SLAM. *IEEE Transactions on Robotics*. <https://doi.org/10.1109/TRO.2007.903811>
- Kobayashi, K., Cheok, K. C., & Watanabe, K. (1995). Estimation of absolute vehicle speed using fuzzy logic rule-based Kalman filter. *In Proceedings of the American Control Conference*. <https://doi.org/10.1109/acc.1995.532084>
- Lin, S. Y., & Chen, Y. C. (2011). SLAM and navigation in indoor environments. *In Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*. https://doi.org/10.1007/978-3-642-25367-6_5
- Liu, J. S., & Chen, R. (1998). Sequential monte carlo methods for dynamic systems. *Journal of the American Statistical Association*. <https://doi.org/10.1080/01621459.1998.10473765>
- Montemerlo, M., Thrun, S., Koller, D., & Wegbreit, B. (2002). FastSLAM: A factored solution to the simultaneous localization and mapping problem. *Proceedings of the National Conference on Artificial Intelligence*, 593–598.
- Okawa, Y., & Namerikawa, T. (2013). H^{∞} Filter-Based SLAM with the Observation on an a priori Known Landmark. *SICE Journal of Control, Measurement, and System Integration*. <https://doi.org/10.9746/jcmsi.6.360>
- Othman, N. A., Ahmad, H., & Namerikawa, T. (2015). Sufficient condition for estimation in designing H_{∞} filter-based slam. *Mathematical Problems in Engineering*, 2015. <https://doi.org/10.1155/2015/238131>

- Paz, L. M., & Neira, J. (2006). Optimal local map size for EKF-based SLAM. *IEEE International Conference on Intelligent Robots and Systems*, (1), 5019–5025. <https://doi.org/10.1109/IROS.2006.282529>
- Ratter, A., & Sammut, C. (2015). Fused 2D/3D position tracking for robust SLAM on mobile robots. *IEEE International Conference on Intelligent Robots and Systems*, 2015-Decem, 1962–1969. <https://doi.org/10.1109/IROS.2015.7353635>
- Rongchuan, S., Shugen, M., Bin, L., & Yuechao, W. (2008). Improving consistency of EKF-based SLAM algorithms by using accurate linear approximation. *IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, AIM, (1), 619–624. <https://doi.org/10.1109/AIM.2008.4601731>
- Smith, R. C., & Cheeseman, P. (1986). On the Representation and Estimation of Spatial Uncertainty. *The International Journal of Robotics Research*. <https://doi.org/10.1177/027836498600500404>
- Smith, R., Self, M., & Cheeseman, P. (1988). Estimating Uncertain Spatial Relationships in Robotics. In *Machine Intelligence and Pattern Recognition*. <https://doi.org/10.1016/B978-0-444-70396-5.50042-X>
- Sun, C., Hou, J., Zhang, A., & She, Z. (2017). A novel intelligent particle filter for process monitoring. *7th International Conference on Intelligent Control and Information Processing, ICICIP 2016 - Proceedings*, 91–97. <https://doi.org/10.1109/ICICIP.2016.7885882>
- Thrun, S., Burgard, W., & Fox, D. (2000). Real-time algorithm for mobile robot mapping with applications to multi-robot and 3D mapping. In *Proceedings - IEEE International Conference on Robotics and Automation*. <https://doi.org/10.1109/robot.2000.844077>
- Tobata, Y., Kurazume, R., Iwashita, Y., & Hasegawa, T. (2010). Automatic laser-based geometrical modeling using multiple mobile robots. *2010 IEEE International Conference on Robotics and Biomimetics, ROBIO 2010*, 363–369. <https://doi.org/10.1109/ROBIO.2010.5723354>
- Ullah, I., Su, X., Zhang, X., & Choi, D. (2020). Simultaneous Localization and Mapping Based on Kalman Filter and Extended Kalman Filter, 2020.
- West, M. E., & Syrmos, V. L. (2006). Navigation of an autonomous underwater vehicle (AUV) using robust SLAM. *Proceedings of the IEEE International Conference on Control Applications*, 1801–1806. <https://doi.org/10.1109/CACSD-CCA-ISIC.2006.4776914>
- Yang, F., Wang, Z., Lauria, S., & Liu, X. (2009). Mobile robot localization using robust extended H_∞ filtering. *Proceedings of the Institution of Mechanical Engineers. Part I: Journal of Systems and Control Engineering*, 223(8), 1067–1080. <https://doi.org/10.1243/09596518JSCE791>
- Zhang, Y., Zheng, Y. F., & Luo, Y. (2016). Rao-Blackwellized particle filter SLAM algorithm based on Gaussian distribution resampling [J]. *Control and Decision*, 31(12), 2299–2304.
- Zhang, F., Li, S., Yuan, S., Sun, E., & Zhao, L. (2017). Algorithms Analysis of Mobile Robot SLAM based on Kalman and Particle Filter. *The 9th International Conference on Modelling, Identification and Control (ICMIC 2017), Kunming, China, (Icmic)*, 1050–1055.
- Zhang, S., Wang, Z., Ding, D., Dong, H., Alsaadi, F. E., & Hayat, T. (2016). Nonfragile H_∞ Fuzzy Filtering with Randomly Occurring Gain Variations and Channel Fadings. *IEEE Transactions on Fuzzy Systems*, 24(3), 505–518. <https://doi.org/10.1109/TFUZZ.2015.2446509>
- Zhang, Y., Zheng, Y. F., & Luo, Y. (2016). Rao-Blackwellized particle filter SLAM algorithm based on Gaussian distribution resampling [J]. *Control and Decision*, 31(12), 2299–2304.
- Zhang, F., Li, S., Yuan, S., Sun, E., & Zhao, L. (2017). Algorithms Analysis of Mobile Robot SLAM based on Kalman and Particle Filter. *The 9th International Conference on Modelling, Identification and Control (ICMIC 2017), Kunming, China, (Icmic)*, 1050–1055.
- Zhang, S., Wang, Z., Ding, D., Dong, H., Alsaadi, F. E., & Hayat, T. (2016). Nonfragile H_∞ Fuzzy Filtering with Randomly Occurring Gain Variations and Channel Fadings. *IEEE Transactions on Fuzzy Systems*, 24(3), 505–518. <https://doi.org/10.1109/TFUZZ.2015.2446509>